Lösungen zur Hausaufgabe: "Die Differenzenquotientenfunktion"

a)

$$m_t(4) = \lim_{x \to 4^+} \frac{32 - 2\cdot x^2}{4 - x} = \lim_{n \to \infty} \frac{32 - 2\cdot\left(4 + \frac{1}{n}\right)^2}{4 - \left(4 + \frac{1}{n}\right)} =$$

$$\lim_{n \to \infty} \frac{32 - 2\cdot\left(16 + \frac{8}{n} + \frac{1}{n^2}\right)}{4 - \left(4 + \frac{1}{n}\right)} = \lim_{n \to \infty} \frac{32 - 32 - \frac{16}{n} - \frac{2}{n^2}}{\frac{-1}{n}} =$$

$$\lim_{n \to \infty} \frac{\frac{1}{n}\cdot\left(-16 - \frac{2}{n}\right)}{\frac{1}{n}\cdot(-1)} = 16$$

$$m_t(4) = 16$$

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b)

$$m_t(x_0) = \lim_{x \to x_0^+} \frac{2\cdot x_0^2 - 2\cdot x^2}{x_0 - x} = \lim_{n \to \infty} \frac{2\cdot x_0^2 - 2\cdot\left(x_0 + \frac{1}{n}\right)^2}{x_0 - \left(x_0 + \frac{1}{n}\right)} =$$

$$\lim_{n \to \infty} \frac{2\cdot x_0^2 - 2\cdot\left(x_0^2 + \frac{2\cdot x_0}{n} + \frac{1}{n^2}\right)}{x_0 - \left(x_0 + \frac{1}{n}\right)} = \lim_{n \to \infty} \frac{2\cdot x_0^2 - 2\cdot x_0^2 - \frac{4\cdot x_0}{n} - \frac{2}{n^2}}{\frac{-1}{n}} =$$

$$\lim_{n \to \infty} \frac{\frac{1}{n}\cdot\left(-4\cdot x_0 - \frac{2}{n}\right)}{\frac{1}{n}\cdot(-1)} = 2\cdot x_0$$

$$m_t(x_0) = 4\cdot x_0$$

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